Sec. 4.4 Applications to Compound Interest

Principal – the total amount borrowed or saved at the beginning of the term (starting amount)

Rate of interest – expressed as a percent, the amount charged for use of the principal for a given period of time (term of contract)

Simple Interest Formula – when the principal P is borrowed for a period of t years at a annual interest rate r (as a decimal) is

$$I = Prt$$

Payment period – common contracted time setups including

a. annually – once per year
b. semiannually – twice per year
c. quarterly – four times per year
d. monthly – twelve times per year
e. daily – 365 times per year

Compounded Interest – when the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on a new principal amount

$$A = P(1+\frac{r}{n})^{nt}$$

where A is the amount after t years from principal P invested at an annual interest rate r compounded n times per year.

Ex. A credit union pays interest at 8% per annum compounded quarterly on a certain savings plan. If $1000 is deposited in such a plan and the interest is left to accumulate, how much is in the account after 8 years?

$$A = 1000\left(1 + \frac{.08}{4}\right)^{4(8)}$$

$$= 1000(1.02)^{32}$$

$$A = \$1884.54$$

Ex. Compare an investment of $2000 at an annual rate of 10% after 3 years if it is compounded annually, quarterly, monthly, and daily. What do you notice about the amounts? Why do you think this happens?

$A = 2000\left(1 + \frac{.10}{1}\right)^{3}$ $A = 2000\left(1 + \frac{.10}{4}\right)^{3 \cdot 4}$ $A = 2000\left(1 + \frac{.10}{12}\right)^{3 \cdot 12}$ $A = 2000\left(1 + \frac{.10}{365}\right)^{3 \cdot 365}$

$A = \$2662$ $A = \$2689.78$ $A = \$2696.36$ $A = \$2699.61$

Nominal versus Effective Rate:

The expression 12% compounded monthly means that interest is added twelve times per year and that 12%/12 = 1% of the current balance is added each time. We refer to the 12% as the nominal rate (nominal means "in name only").

When the interest is compounded more frequently than once a year, the account effectively earns more than the nominal rate. Thus, we distinguish between nominal rate and effective annual rate, or effective rate. The effective annual rate tells you how much interest the investment actually earns. In the US, the effective annual rate is sometimes called the APY (annual percentage yield).

Ex. What is the effective annual rate of an account that pays interest at the nominal rate of 6% per year, compounded daily? Compounded hourly?

$A = P\left(1+\frac{.06}{365}\right)^{365}$

$A = P(1.0618313)$

$r = 6.18313\%$

$A = P\left(1+\frac{.06}{365\times 24}\right)^{365\times 24}$

$A = P(1.0618363)$

$r = 6.18363\%$

Ex. At the beginning of the year you deposit P dollars in an account paying interest at the nominal rate of 4% per year compounded quarterly. By what factor does P grow in 3 years?

$A = P\left(1+\frac{.04}{4}\right)^{3\cdot 4}$

$= P(1+.01)^{12}$

$= P(1.01)^{12}$

$= P(1.126825)$

Growth Factor: 1.1268

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